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Predictive models used to estimate the resilient modulus of subgrade soil for pavement design

Modelos preditivos utilizados para estimar o módulo resiliente do solo de subleito para o dimensionamento de pavimentos

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ABSTRACT
Obtaining the resilient modulus (MR) of the subgrade soils to be used in mechanistic-empirical pavement design models, such as MeDiNa, is a complex and expensive process, due to the high costs of acquiring and operating the repeated triaxial load testing equipment. This study investigated the use of Artificial Neural Networks (ANN) to create models for the prediction of the MR of soils based on their index properties. A database was created for the modeling with experimental datasets obtained from the state of Ceará, Brazil. The results indicated that the ANN is able to predict the MR of the subgrade soil very accurately (with a correlation of 0.9878 for the test dataset. These results were used to generate estimates that can be included in an integrated approach to the mechanistic-empirical pavement design in Brazil (MeDiNa), which reduces both the financial costs and the execution time of projects, especially for the design of low-volume roads.

RESUMO
A obtenção do módulo de resiliência (MR) dos solos e do subleito para uso em dimensionamento mecanístico-empírico de pavimentos, como o MeDiNa, é um processo complexo e oneroso, devido aos elevados custos de aquisição e operação dos equipamentos triaxiais de cargas repetidas. Este estudo investigou o uso de redes neurais artificiais (RNAs) para a criação de modelos de previsão do MR dos solos com base em suas propriedades índices. Foi criado um banco de dados para a modelagem a partir de conjuntos de dados experimentais, obtidos no estado do Ceará, Brasil. Os resultados indicaram que as RNAs são capazes de prever o MR dos solos com baixo erro (com correlação de 0,9878 para o conjunto de dados de teste). Esses resultados foram utilizados para gerar estimativas que podem ser incluídas em uma abordagem integrada ao método de dimensionamento mecanístico-empírico de pavimentos no Brasil (MeDiNa), reduzindo tanto os custos financeiros quanto o tempo de execução dos projetos, especialmente para o dimensionamento de rodovias de baixo volume de tráfego.

1. Introduction

In pavement and geotechnical engineering, it is common to seek a method for determining the mechanical properties of granular materials and to conduct studies to enhance their behavior as a foundation, subgrade, and pavement layers, such as base and subbase, as found in: Liu *et al.* (2014); Osouli *et al.* (2021); Singh, Trivedi & Shukla (2019); Esmaeili *et al.* (2016) and Dhar & Hussain (2021). Similarly, many researchers have developed models to estimate and describe empirical and mechanical geotechnical parameters, such as degree of saturation, plasticity, California Bearing Ratio (CBR), Permanent Deformation (PD), and resilient modulus (MR), for use in pavement structure design (Souza, Ribeiro & Silva, 2022; Rahman & Erlingsson, 2015; Ramos *et al.*, 2020; Khasawneh & Al-Jamal, 2019).

In Brazil, it is still common practice to use empirical methods for the design of pavements (Souza, Ribeiro & Silva, 2022; Silva *et al.*, 2023; Lacerda *et al.*, 2024), based on the CBR. Singh, Trivedi & Shukla (2019) explains that conducting the CBR test helps in the qualification and adequacy of projects with respect to mechanical properties. In addition, this parameter implies a primary evaluation regarding soil intervention and correction, providing a basis for the decision-making process in the adjustment of pavement projects, in order to enable lower costs and greater efficiency. Efforts are nevertheless being made to introduce a mechanistic-empirical method of asphaltic pavement design in Brazil, using the MR as the design parameter.

Mathematically speaking, the MR (Equation 1) is defined as the ratio between the repeated deviator stress (σ_d) and the recoverable axial strain (ε_r). In the laboratory, the MR can be determined by a repeated load triaxial compression test, which is regulated in Brazil by DNIT standard 134/2018-ME (DNIT, 2018).

$$\mathrm{MR} = \frac{\sigma_d}{\varepsilon_r}, \tag{1}$$

where MR = resilient modulus; $\sigma_d = \sigma_1 - \sigma_3$ = deviator stress; σ_1 = major stress; σ_3 = confining stress; ε_r = recoverable strain.

In Brazil, attempts to develop a mechanistic-empirical pavement design method have faced a number of challenges, primarily for the implementation of the MR assays. This is due in particular to the lack of qualified personnel and the cost of the triaxial repeated stress apparatus, which is priced at around US$ 80,000. According to Ribeiro, Silva & Barroso (2018) and Lacerda *et al.* (2024), in Brazil there has been limited availability of this type of equipment, which is mainly found in research centers and universities located in the southern region of the country. Despite this limitation, there is a growing trend toward expanding triaxial testing studies, as established and reinforced by DNIT Standard 134/2010 and its revision DNIT 134/2018-ME (DNIT, 2010, 2018). These standards provide a basis for expanding access to this type of testing equipment, encouraged by companies operating in the geotechnical analysis sector. In addition, the need to intensify the prediction of empirical methods that provide a basis for mechanical studies becomes evident, given the nonconformities found in the field. It also remains feasible to use the design of subgrade or sub-base layers through methods commonly employed to investigate the longitudinal and vertical soil structure, based on CBR, shear strength, and compressive strength, as proposed in the studies by Singh, Trivedi & Shukla (2019), Esmaeili *et al.* (2016), and Dhar & Hussain (2021). Given this scenario, one viable alternative would be to use models to predict the MR, based on values obtained in more simpler tests, such as those suggested by Rahim (2005), Nazzal & Tatari (2013), Sadrossadat, Ali & Saeedeh (2016) and Mousavi, Gabr & Borden (2018).

The development of these models can contribute to the implementation of highway projects through the cost-effective and more rational production of data and parameters on the more complex geotechnical characteristics of the subgrade soils (Taskiran, 2010; Yildirim & Gunaydin, 2011; Alawi & Rajab, 2013; Nguyen & Mohajerani, 2017). A number of studies, such as those of Rahim (2005), Ribeiro, Silva & Barroso (2015), Sadrossadat, Ali & Saeedeh (2016) and Li & Wang (2019), have pointed out that a range of design approaches, such as the Guide for Mechanistic-Empirical Design of New Rehabilitated Pavement Structures, the NCHRP 1-37A (NCHRP, 2004), and the ME-PDG –Mechanistic-Empirical Pavement Design Guide (AASHTO, 2008), permit the use of MR estimates based on the alternative properties of the materials. Ghanizadeh, Heidarabadizadeh & Heravi (2021), Baghbani *et al.* (2022), and Tanyildizi & Gökalp (2025) corroborate that the use of Artificial Intelligence (AI) as a predictive model to support feasibility studies for correlating laboratory and field data facilitates the development of a parallel and dynamic system. This approach allows unrestricted and topic-specific interpretation of the computational model and the real spectrum of the field of investigation. Of the techniques used for this modeling, ANN have proved to be the most successful for the prediction of the geotechnical variables, and have been widely adopted in highway engineering and geotechnical projects.

ANN are based on an AI approach that attempts to simulate human brain function in a simplified manner in a computer system. This approach is based on the use of parallel systems composed of simple processing units (neurons) that calculate specific (non-linear) mathematical functions. These units are arranged in one or more layers interlinked through a large number of generally unidirectional connections (Haykin, 2007).

The ANN most used in geotechnical engineering is the Multilayer Perceptron (MLP) (Zhang & Yu, 2016), which is made up of multiple layers of neurons, arranged in three types: (*i*) the input layer, which receives the external stimuli; (*ii*) one or more intermediate (hidden) layers that amplify the capacity of the ANN to capture the behavior of the most complex characteristic of the phenomenon being modeled, and (*iii*) the output layer, which presents the responses to the stimuli presented to the ANN. The MLP thus consists of multiple layers of neurons that simulate the function of biological neurons (Sitton, Zeinali & Story, 2017). The inputs are multiplied by a synaptic weight that is, initially, random. These weighted inputs are then added and increased to modify the output of the neuron. The output value is normalized through the application of an activation function. Equations 2, 3 and 4 show a non-linear mathematical model of an artificial neuron:

$$u_k = \sum_{i=1}^{m} w_{ki} x_i + b_k \tag{2}$$

$$v_k = u_k + b_k \tag{3}$$

$$y_k = f(v_k) \tag{4}$$

where x_m = ANN input; w_{ki} = synaptic weights; b_k = bias term; u_k = linear combination of input signals; $f(v_k)$ = activation function, and y_k = neuron output.

One of the most interesting properties of an ANN is its capacity to learn from the examples presented to it, and thus improve its performance through a continuous training process, as proposed in the studies by Ghanizadeh, Heidarabadizadeh & Heravi (2021), Ghanizadeh & Delaram (2021), Ghanizadeh & Heidarabadizadeh (2024) and Heidarabadizadeh, Ghanizadeh & Behnood (2021), this framework explains the efficiency process to which ANN methods tend to contribute. These methods are not considered the sole reference, but rather determinant in correlation measurements through the coefficient of determination (R^2), based on data constructed from a database. They are used to evaluate the ability to analyze subgrade and sub-base layers, with or without interference from soil physical property corrections, and are mainly applied to obtain MR values that ensure the quality of pavement structures subjected to dynamic loads and deformations. The network training is based on the modification of all the existing synaptic weights and thresholds, based on the experience gained about the phenomenon under analysis, which is typically available in a dataset that contains pairs of known inputs and outputs (Ribeiro, Silva & Barroso, 2018).

The majority of learning algorithms use three types of dataset (training, validation, and testing), in order to avoid overfitting and underfitting. Based on the three datasets, the training stopping rule of the Levenberg–Marquardt algorithm was applied. This rule prevents the network from continuing the training process when the validation dataset presents a certain number of consecutive increases in error. In this study, six consecutive validation cycles without a significant reduction in error were adopted as the stopping criterion, which triggered the interruption of the training process. The training and validation datasets are used in the supervised learning phase of the ANN, while the testing dataset is used to test the knowledge level of the network. Once trained, it is possible to use the synaptic weights to calculate an output based on input data that had never been presented to the neural network. For this, it is only necessary to apply the weights exported from the model in Equations 2, 3 and 4, for example, in a computer spreadsheet.

A number of studies have demonstrated the success of ANN for the prediction of models of the mechanical characteristics of subgrade soils (Rakesh *et al.*, 2006; Park, Kweon & Lee, 2009; Zhang & Yu, 2016). In other cases, the ANN have been combined with other machine learning techniques to predict geotechnical characteristics (Gunaydin, Gokoglu & Fener, 2010; Nazzal & Tatari, 2013; Erzin & Turkoz, 2016; Tenpe & Patel, 2018; Pavini, 2002; Mollahasani *et al.*, 2010).

Zeghal & Khogali (2005) successfully predicted the MR of granular materials using ANN, with the input data: density, stress states, and humidity of the material. Zaman, Chen & Laguros (1994) developed a range of models based on ANN to predict the resilient behavior of the subgrade soil based on stress states and property indices, used an input variables. Johari, Javadi & Habibagahi (2011) combined ANN and Genetic Algorithms (GA) to model the results of the triaxial tests based on explanatory data, such as density, the axial deformation, the degree of saturation, the deviator stress, and the confinement strength. Kim, Yang & Jeong (2014) used property indices and stress states to estimate the MR of nine different types of subgrade in the American state of Georgia using an ANN as a modeling tool. Zhang & Yu (2016) used a backpropagation type of model to predict the resilient modulus of the subgrade of highways in Harbin, China, meanwhile, Ghanizadeh, Heidarabadizadeh & Heravi (2021) used Gaussian Process Regression (GPR) modeling to predict the resilient modulus characteristics of chemically stabilized materials.

In general, the development of an ANN model requires the selection of a data training procedure, the choice of an adequate architecture, and a careful and thorough process of training and validation (Boruvka & Penizek, 2007). As mentioned above, ANN have been used successfully to predict geotechnical characteristics in pavement applications. The largest part of these models has been developed without datasets provide from Brazil. It's possible to observe that the previous studies reported in this paper were created for each area in a determined country. This affirmation reinforces the necessity to validate models to different studies areas using their datasets, as recommended by Souza, Ribeiro & Silva (2022), Sadrossadat, Ali & Saeedeh (2016), Patel & Desai (2010), and Zumrawi (2012).

Given this, the present study proposes an approach for the creation of models for the prediction of the MR using ANN and the indices of soil properties from the state of Ceará, in northeastern Brazil. This study is distinguished by the size and consistency of its database, comprising 1,341 resilient modulus (MR) values obtained from standardized MR testing conducted within a single state. For each soil specimen, up to 18 MR values were determined as a function of the applied stress state. The dataset includes 75 soil samples collected in the state of Ceará. This dataset allows for a robust and detailed regional representation. Furthermore, estimating MR based on easily obtainable parameters (such as sieve granulometry and compaction data) increases the practicality and immediate applicability of the proposed approach in highway engineering. In addition, these data from Ceará may be incorporated into a national MR database, enabling the development of more generalized artificial intelligence models for the entire country. This would support their application in different regions and strengthen predictive modeling at the national level for use in the MeDiNa pavement design method, particularly in situations where stiffness test data are unavailable or when laboratory testing cannot be performed.

2. Materials and methods

2.1. Database and analysis

A considerable volume of data is necessary to construct predictive models based on an ANN. The larger and more representative the number of examples of the variable to be modeled, the greater the probability for the generalization of the neural model (Sitton, Zeinali & Story, 2017; Ribeiro, Silva & Barroso, 2015). Given this, the application, testing, and validation of the predictive MR models for the Brazilian state of Ceará, a comprehensive database was compiled from the collection and laboratory analysis of all the soil groups found in the study region.

The database includes 1341 samples with 15 variables: liquid limit (LL); plasticity index (PI); percentage of the soil particles passing through sieves of different sizes (#1", #3/8", #4, #10, #40, and #200); AASHTO classification (class); maximum dry density (γ_{dmax}); optimum moisture content (w_{opt}); CBR value; expansion (exp); confining stress (σ_3); deviator stress (σ_d); and MR. All data were obtained following standard Brazilian procedures, including grain size distribution determined in accordance with NBR 7181 (ABNT, 1984) and Proctor compaction tests (ABNT, 2019); plasticity limit (ABNT, 2016b); liquid limit (ABNT, 2016a); the determination of the CBR (ABNT, 2016c), and the determination of the resilient modulus (DNIT, 2018). The compaction tests were conducted at standard energy for part of the samples and at intermediate energy levels for the remaining ones. Overall, the present study included soils of nine AASHTO classes (A-1-a, A-1-b, A-3, A-2-4, A-2-6, A-4, A-5, A-6, and A-7-5) from all parts of the Brazilian state of Ceará.

Table 1 presents an overview of the variables included in the database. A generalized linear model, with a 95% confidence limit, was applied to determine the MR default values for each AASHTO class (Table 2). Dai & Zollars (2002), Kim & Kim (2007), Pal & Deswal (2014), Kim, Yang & Jeong (2014) and (Sadrossadat, Ali & Ghorbani, 2018) used some of the variables included in the present study (Table 1) in their soil MR prediction models. However, much smaller databases were used in these previous studies, in comparison with that analyzed in the present study.

2.2. Modeling of the resilient modulus

The Neural Network Tool, run in MATLAB version 2015, was used to model the data in the present study. The initial step consisted of the compilation of a test tree that consisted of a battery of tests used to determine the most adequate architecture for the generation of the estimates of the phenomenon being modeled. This most adequate architecture was that for which the errors of the estimated outputs, for the full set of tests, were the lowest in comparison with the real values. This tool has been used in a number of ANN modeling studies. In particular, Ribeiro, Silva & Barroso (2015) supported the use of this modeling tool due to its enormous flexibility for the formatting and manipulation of files, and the diversity of algorithms that can be implemented with a high level of efficiency.

The data used in the present study for model training were divided into three sets: (*i*) 60% of the data were used for training; (*ii*) 20% were used for validation, which is necessary for the implementation of the stop rule of the learning algorithm, and (*iii*) 20% of the data were used for testing as suggested in Haykin (2007), Ribeiro, Silva & Barroso (2018), and Souza, Ribeiro & Silva (2022). These datasets were selected randomly and were not repeated.

Table 1 • Descriptive statistics of the variables that make up the dataset used in the present study

Parameter	Mean	Median	Mode	Std. dev.	Range
w_{opt} (%)	9.33	9.20	7.70	2.58	12.80
γ_{dmax} (g/cm^3)	2.00	2.01	2.07	0.13	0.54
CBR (%)	38.04	32.00	11.00	28.06	110.20
exp (%)	0.10	0.00	0.00	0.19	0.80
LL (%)	12.76	16.00	0.00	13.08	46.00
PI (%)	3.88	3.00	0.00	4.35	14.00
#1” (%)	94.00	97.00	100.00	7.42	20.00
#3/8” (%)	78.42	79.00	100.00	21.83	60.00
#4 (%)	72.40	74.00	100.00	27.58	78.00
#10 (%)	68.49	68.00	98.00	28.68	82.00
#40 (%)	52.37	54.00	20.00	25.95	75.00
#200 (%)	20.96	13.00	13.00	15.82	66.00
σ_3 (MPa)	0.07	0.05	0.05	0.04	0.12
σ_d (MPa)	0.14	0.10	0.10	0.10	0.39
MR (MPa)	542.38	509.00	635.00	372.91	1987.93

Table 2 • Generalized linear model of the MR default values recorded for each AASHTO class

AASHTO Symbol	MR Default (MPa)	MR Range (MPa)	Percent in sample (%)
A-5	109.00	93 – 124	1.34
A-3	112.00	102 – 122	2.68
A-4	234.00	216 – 253	14.47
A-6	311.00	274 – 348	5.29
A-2-4	421.00	388 – 453	38.87
A-2-6	515.00	439 – 591	8.05
A-7-5	589.00	536 – 643	1.34
A-1-a	779.00	743 – 814	12.08
A-1-b	829.00	793 – 865	15.88

Modeling is used to obtain a relatively simple approach to pavement design, using variables that can be obtained easily in the laboratory. For this, the ANN input data (LL, PI, #1”, #3/8”, #4, #10, #40, #200, class, γ_{dmax}, w_{opt}, CBR, exp, σ_3 and σ_d) were tested initially to determine how they are related. Some of these variables were then omitted recurrently, with the predictor variables or ANN input data being alternated to determine which model performed best for the prediction of the MR. A number of different algorithms, with varying parameters (the number of intermediate layers, the number of neurons per layer, learning rates, the momentum term, and the number of training periods) were also tested.

The definition of the number of neurons in the hidden layer generally follows a trial-and-error approach; however, some studies propose initial estimation methods to support this decision. In this study, the method proposed by Hecht-Nielsen (1989) was adopted, in which the hidden layer should contain approximately twice the number of input neurons plus one additional neuron.

The Levenberg-Marquardt (TRAINLM) algorithm was selected to initiate the tests. This algorithm is a modified form of the backpropagation error algorithm. This algorithm was chosen for the initial testing based on the recommendations of Beale, Hagan & Demuth (2010). The TRAINLM is a function that updates the weights and values of the biases based on their optimization. This algorithm is widely considered to be the most rapid backpropagation error training algorithms, although it does require more computer memory than other algorithms (Zhang & Yu, 2016; Ribeiro, Silva & Barroso, 2018; Tenpe & Patel, 2018). Beale, Hagan & Demuth (2010) recommended TRAINLM as a rapid training algorithm for moderately-sized datasets, as well as having good potential for generalization, in most cases. Although other algorithms, such as the backpropagation error descending gradient algorithm with momentum and an adaptive learning rate (TRAINGDX) have been tested previously, the Levenberg-Marquardt algorithm has been proven to be the most efficient.

2.3. Structural evaluation using ANN-estimated MR

To verify the reliability of the neural network under real traffic conditions, simulations were performed using the MeDiNa software. The network with the best statistical performance was selected for simulation, following a methodology similar to that adopted by Lacerda *et al.* (2024). The analyses focused on Wheel Track Rutting (WTR) and the percentage of cracked area. The parameters adopted in the pavement design simulations were based on principal arterial roads, considering both a linear MR model (average MR value) and a nonlinear constitutive model for MR proposed by Pezo (1993). The adopted traffic data are presented in Table 3.

The designed pavement structure consists of four layers. The surface course, obtained from the MeDiNa database, has a thickness of 10 cm and consists of Class 1 asphalt concrete, with an MR of 5764 MPa. The base layer is composed of a soil–aggregate material extracted from Bastos (2013), with a thickness of 15 cm and a linear MR of 506 MPa. Finally, the subbase layer (15 cm thick) and the subgrade correspond to soil samples adopted from the database, whose MR and PD parameters are presented in Table 4.

The PD parameters were adapted from the MeDiNa database, considering materials with similar characteristics, such as compaction parameters (γ_{dmax} and w_{opt}) and grain size distribution. In this case, parameters corresponding to a non-lateritic sandy-silty soil (NA’) were adopted.

Table 3 • MeDiNa traffic data inputs

Parameter	Value
Average Daily Traffic (1st year)	327
Vehicle factor (1st year)	3417
Annual number of standard axle load applications (1st year)	4.08E+05
Percentage of vehicles in the design lane	100
Annual number of standard axle load applications in the design lane	3
Design period (years)	10
Total number of standard axle load applications in the pedestrian lane	4.68E+06

Table 4 • Parameters related to MR and PD of the samples used

Sample	Constitutive Models	Constants	Subgrade	Sub-base
MR with values calculated by ANN (calculated MR)	Nonlinear	k1	444.918	699.682
		k2	0.585	0.191
		k3	−0.398	−0.121
		R^2	0.708	0.740
	MR linear		210.111	535.056
MR with laboratory values (real MR)	Nonlinear	k1	446.149	860.900
		k2	0.433	0.267
		k3	−0.214	−0.129
		R^2	0.370	1.000
	MR linear		212.556	540.176
MR constants used in the base layer (Bastos, 2013)	Nonlinear	k1	1161.36	
		k2	0.53	
		k3	−0.296	
	MR linear		506	
PD constants used in the base layer		$\psi 1$	0.31	
		$\psi 2$	0.06	
		$\psi 3$	0.85	
		$\psi 4$	0.05	
PD constants used in the sub-base layer and subgrade		$\psi 1$	0.57	
		$\psi 2$	0.71	
		$\psi 3$	0.27	
		$\psi 4$	0.05	

For the structural analysis in MeDiNa, the predictive model with the best performance for estimating the resilient modulus (MR) was adopted. Simulations were carried out considering two distinct soils selected from the database used in this study, one assigned to the subgrade layer and the other to the subbase layer, composing a single pavement structure. For these same materials, two sets of simulations were performed: one using MR values obtained experimentally in the laboratory and another using MR values predicted by the selected model. In addition, both linear and non-linear material behavior were considered, resulting in a total of four simulation scenarios. This approach enabled a direct assessment of the impact of replacing experimental MR values with predicted values on pavement structural performance, while keeping all other analysis conditions constant.

3. Results and discussion

The Levenberg-Marquardt algorithm was the most appropriate backpropagation error procedure for the prediction of the MR. This algorithm produced the best results for the prediction of the parameters expected as ANN outputs from the dataset. The linear coefficient of correlation (R), mean of squared error (MSE), and mean absolute error (MAE) were used to evaluate the performance of the different models. Equations 5, 6 and 7 show the performance measures R, MSE and MAE, respectively:

$$R = \frac{\sum_{i=1}^{n}(h_i - \bar{h}_i)(t_i - \bar{t}_i)}{\sqrt{\sum_{i=1}^{n}(h_i - \bar{h}_i)^2 \sum_{i=1}^{n}(t_i - \bar{t}_i)^2}} \tag{5}$$

$$MSE = \frac{\sum_{i=1}^{n}(h_i - t_i)^2}{n} \tag{6}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}(h_i - t_i) \tag{7}$$

where, h_i and t_i are the actual and predicted output values, respectively, for the i-th output, respectively, $\bar{h}_i$ and $\bar{t}_i$ are the average of the actual and predicted outputs, and n is the number of samples.

For the purposes of the present study, only the five best models (M1–M5) identified in the analyses for the prediction of the MR of the soils of the Brazilian state of Ceará are presented here (Table 5). The results of the different topologies vary principally in the input data and the number of neurons in the intermediate layer, while the MR values are the output variables for all the models (Table 5). It is important to note that, for each MR, prediction model (M1–M5), the neural network structure consisted of input neurons, a hidden layer, and an output neuron, respectively, R, MSE, and MAE values are presented for the training, validation, and testing datasets (Table 6).

Table 5 • The variables of the best neural models identified in the present study

Model	Input	Output
M1	LL, PI, #1", #3/8", #4, #10, #40, #200, Class, γ_{dmax}, w_{opt}, CBR, exp, σ_3, σ_d	
M2	LL, PI, #1", #3/8", #4, #10, #40, #200, class, γ_{dmax}, w_{opt}, σ_3, σ_d	
M3	#1", #3/8", #4, #10, #40, #200, class, γ_{dmax}, CBR, σ_3, σ_d	MR
M4	#1", #3/8", #4, #10, #40, #200, γ_{dmax}, w_{opt}, σ_3, σ_d	
M5	#10, #40, #200, γ_{dmax}, w_{opt}, σ_3, σ_d	

Table 6 • Five best structures obtained with reported errors for the prediction of the MR

Model	M1	M2	M3	M4	M5
Structure	15:31:1	13:27:1	11:23:1	10:21:1	7:15:1
R Training	0.9923	0.9934	0.9896	0.9914	0.9840
R Validation	0.9842	0.9881	0.9827	0.9850	0.9857
R Test	0.9844	0.9847	0.9857	0.9878	0.9847
MSE Training	0.002	0.002	0.003	0.002	0.004
MSE Validation	0.005	0.004	0.005	0.005	0.003
MSE Test	0.003	0.004	0.004	0.003	0.004
MAE Training	0.46	0.35	0.33	0.27	0.3
MAE Validation	12.92	12.57	9.52	7.15	4.87
MAE Test	0.64	2.11	4.89	1.35	7.11

As shown here (Table 4), the best performing models used only a single hidden layer, as predicted by Hecht-Nielsen (1989), Cybenko (1989), and Bounds *et al.* (1988). Model M1 has 15 neurons in the input layer, 31 neurons in the intermediate layer, and one in the output layer (15:31:1). Model M2 has 13 neurons in the input layer, 27 neurons in the intermediate layer, and one in the output layer (13:27:1). Model M3 has 11 neurons in the input layer, 23 neurons in the intermediate layer, and one in the output layer (11:23:1). Model M4 has 10 neurons in the input layer, 21 neurons in the intermediate layer, and one in the output layer (10:21:1). Model M5 has seven neurons in the input layer, 15 neurons in the intermediate layer, and one in the output layer (7:15:1). All the five most efficient models identified in the present study used the identity activation function in the output layer and the sigmoidal tangent as the activation function in the intermediate layer, as shown in Equation 8, with a $[-1; +1]$ activation interval.

$$\varphi = \frac{2}{[(1+e)^{-2v_k}] - 1} \tag{8}$$

The five neural models that produced the best results for the estimation of the MR, presented here, performed well for the measures adopted in the study. In fact, the R, MSE, and MAE values indicated low levels of error for the training, validation, and test datasets of all the models tested (Table 6). It is also important to note that the convergence times of all five models were less than 20 seconds, which can be considered an excellent performance for the amount of data evaluated. This was due to the application of a training stop rule for the Levenberg-Marquardt algorithm, which reduces the accumulation of overfitting, and improves the potential for the generalization of the models.

The success rates of neural models are generally evaluated using the test dataset, given that neural networks are unable to recognize the outputs for the new data presented. These ANN outputs are compared with the real data presented to the ANN only after training and validation. Based on this analysis, it is possible to conclude that model M4 had the best performance for the analysis of the test dataset ($MSE = 0.003$, $R = 0.9878$ and $MAE = 1.35$), and was considered, statistically, as the optimally adjusted model. In addition to having the best statistical performance, model M4 demands the least laboratory work to obtain the input variables, requiring only the data from the particle size analysis and the Proctor compaction tests to obtain the MR. Table 6 shows the variables each model needs to preview MR, so the models M1, M2, and M3 require more laboratory tests than models M4 and M5 which necessity of the same tests.

Models M4 and M5 presented very similar R values, indicating a strong linear association between the observed and predicted values. This result is relevant because a high R value suggests that the behavior of MR may also be satisfactorily described using a linear regression approach. However, since artificial neural networks (ANNs) are inherently capable of capturing complex nonlinear relationships, it is essential to carefully evaluate the error metrics associated with the cost functions, such as MSE and MAE. In this context, Model M4 presented lower error values (MSE and MAE), indicating a smaller discrepancy between the predicted and experimental values.

The coefficients recorded for model M4 were higher than those recorded in previous studies (Sadrossadat, Ali & Saeedeh, 2016; Sadrossadat, Ali & Ghorbani, 2018; Erzin & Turkoz, 2016; Yildirim & Gunaydin, 2011; Taskiran, 2010; Kayadelen *et al.*, 2009), which indicates that the input variables selected for this model are adequate for the prediction of the MR. In this case, it is possible to indicate that model M4 is a function of the parameters presented in Equation 9, with the architecture shown in Figure 1.

$$MR = f(\#1\text{”}, \#3/8\text{”}, \#4, \#10, \#40, \#200, \gamma_{\text{dmax}}, w_{\text{opt}}, \sigma_3, \sigma_d) \tag{9}$$

Model M4 was selected for generalization and recommended for application in the Brazilian state of Ceará. Given this, the adjustment of the experimental MR values to the values obtained by the M4 model for the R, MSE, and MAE of the training, validation, and test datasets are shown in Figures 2, 3 and 4 with their respective measures of performance.

These plots (Figures 2, 3 and 4), together with the measures of model precision indicate that overfitting was avoided, based on the proximity of the MSE and R values of the training, validation, and test datasets, and that the M4 model has good potential for generalization for the production of MR estimates.

Figure 5 presents two curves, representing the predicted and experimental MR values, from the smallest to the largest. This analysis shows clearly that the values calculated by the model (M4) and those produced in the laboratory follow the same general tendency, with a certain degree of difference. Figure 6 shows the residual errors calculated for each of the values in the two curves shown in Figure 5, as well as the minimum and maximum residual errors.

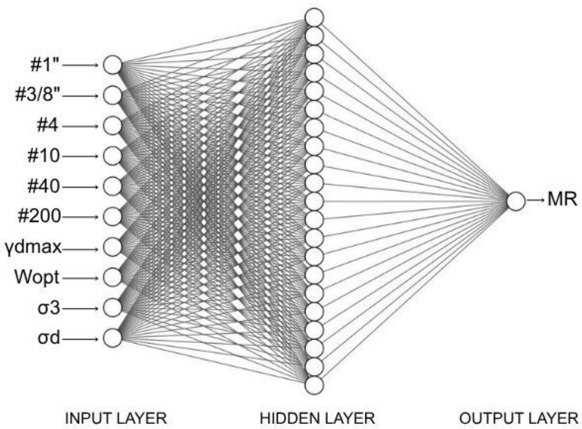

Figure 1 • Architecture of the ANN of the best model (M4)

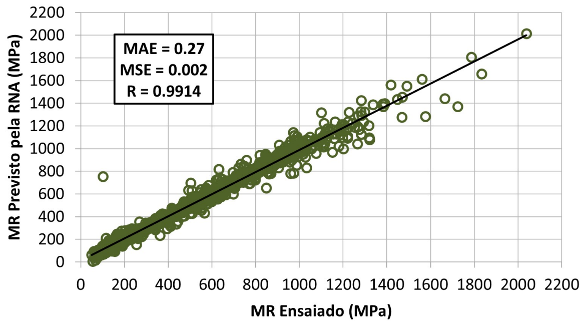

Figure 2 • Plot of the predicted vs. experimental values for the training dataset of M4

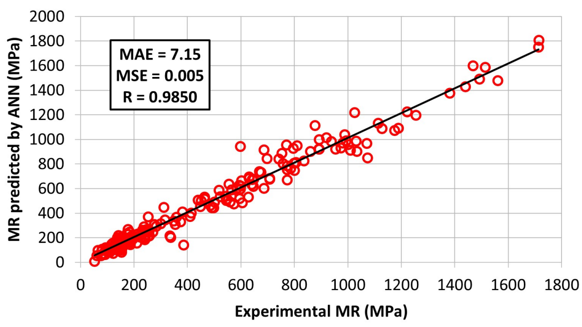

Figure 3 • Plot of the predicted vs. experimental values for the validation dataset of M4

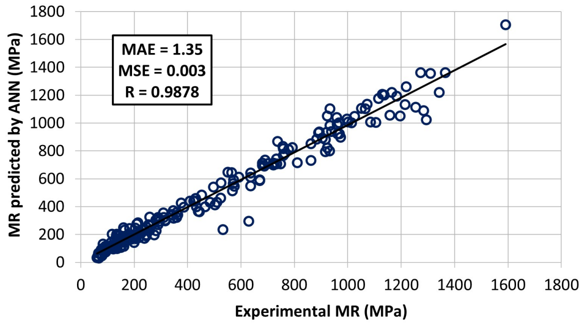

Figure 4 • Plot of the predicted vs. experimental values for the test dataset of M4

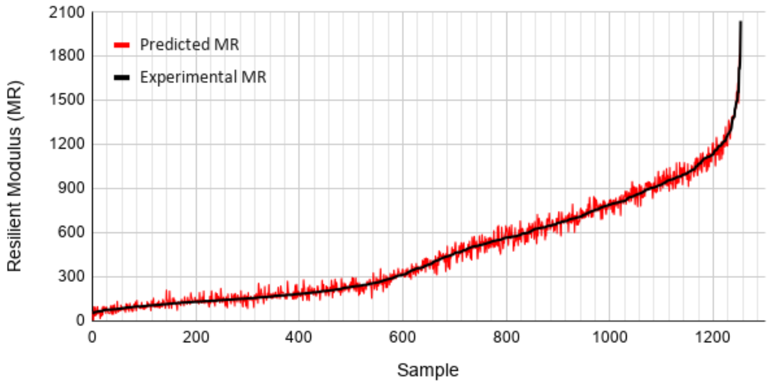

Figure 5 • Predicted and experimental MR values organized in ascending order

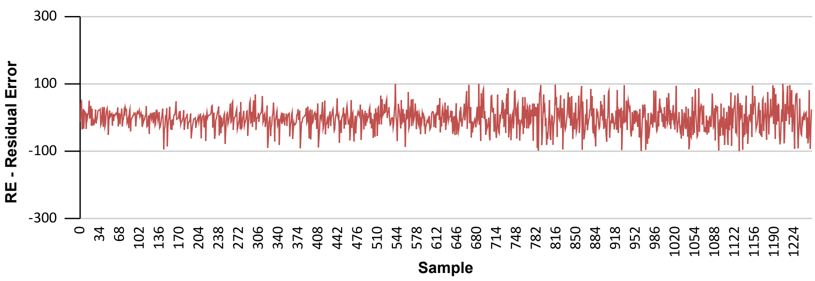

Figure 6 • The residual errors of the experimental and predicted MR values

The distribution of the errors of the predictions of the MR value by model M4 is also shown in Figure 7. This plot shows a high frequency of errors distributed in the lower intervals, which indicates a precise measurement capability and good potential for the generalization of this model.

When analyzed together, Figures 5, 6 and 7 confirm that model M4 has a high level of accuracy for the prediction of the MR of subgrade soils. This further reinforces the conclusion that the model can be generalized for the Brazilian state of Ceará. To support the use of the M4 model, the weights of the intermediate and output layers are shown in Table 7, to enable the implementation of the neural model outside the modeling environment.

Figure 8 presents the WTR results for the simulations carried out, aiming to verify the values under adverse traffic conditions and to compare the linear and nonlinear constitutive MR models, contrasting the actual MR with the calculated MR. The impact on the total WTR values and on the WTR values corresponding to the subgrade layer was analyzed. In addition to the maximum allowable WTR limit of 10 mm for the entire pavement structure, the MeDiNa method also establishes a limit for the subgrade

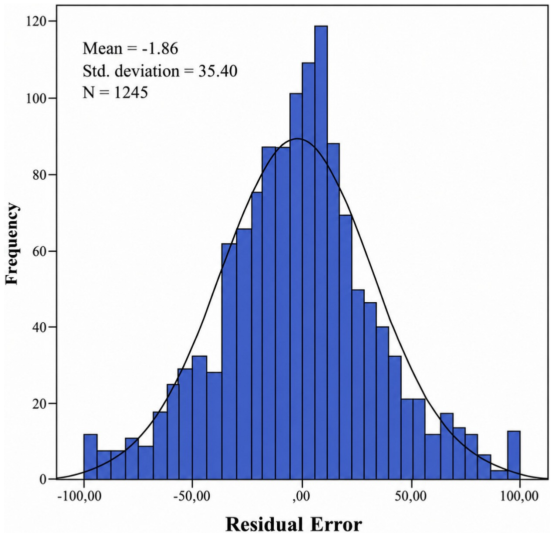

Figure 7 • Histogram of the errors of model M4

Table 7 • The weight and biases of the MR output predicted by model M4

Input layer to hidden layer											Hidden to output layer	
#1"	#3/8"	#4	#10	#40	#200	γ_{dmax}	w_{opt}	σ_3	σ_d	Bias	Weight	Bias
0.46	−0.37	0.58	0.45	−0.58	−0.33	1.13	−1.57	−2.14	−2.55	3.29	0.09	3.36
−3.99	2.59	−2.14	−0.05	−0.27	−4.09	−1.06	−2.63	−0.05	0.14	4.40	−0.74	
1.51	−0.04	−0.84	2.64	0.13	0.33	−2.04	−1.38	0.07	−0.28	−1.50	2.47	
0.40	0.21	−0.78	2.80	−0.86	0.39	−1.71	−0.63	0.07	−0.40	−0.98	−2.08	
−0.25	−0.06	0.38	−1.24	1.03	0.06	−0.50	−0.02	0.03	−0.29	−0.76	5.14	
0.23	−0.77	−2.31	2.57	0.05	−3.00	1.69	2.59	0.13	−0.07	5.48	3.84	
2.21	−2.56	−0.18	1.93	−2.51	−3.43	−0.79	−1.77	−0.15	0.05	0.29	1.17	
−4.05	−2.61	0.16	1.01	−0.58	−1.62	2.11	−0.58	−0.21	0.27	2.53	0.89	
0.68	1.91	−0.56	1.39	−1.08	−0.41	2.40	0.06	0.04	−0.08	−1.36	5.07	
0.95	−1.10	−1.84	1.24	0.36	0.26	−0.81	2.30	−0.12	1.23	−0.90	0.46	
−0.28	1.59	−0.66	2.00	1.63	−1.17	1.69	0.35	0.03	−0.03	−3.39	−3.27	
−1.06	1.92	0.83	2.63	−1.24	−3.67	5.31	−2.20	−0.17	5.42	3.74	0.15	
−2.61	−2.43	−2.90	2.43	2.19	0.35	−3.07	1.11	−0.04	0.12	0.41	2.04	
−1.29	1.69	−1.65	0.46	0.28	−1.43	1.33	−0.94	−0.10	0.57	0.11	0.81	
−0.12	−0.05	−0.20	1.48	−1.70	0.99	0.21	0.19	−0.90	5.64	5.32	−0.57	
1.40	−1.14	−1.56	−0.27	2.24	0.77	−2.17	−0.34	0.33	−2.82	−1.12	−0.17	
0.11	−0.58	−1.30	1.02	0.79	−1.67	0.43	0.45	0.03	−0.26	1.79	−3.52	
−0.29	0.45	0.88	0.08	2.36	2.36	1.30	0.65	0.59	0.43	1.70	0.39	
1.23	1.09	−4.17	1.79	−1.16	5.13	−0.58	−1.16	0.00	0.25	−1.61	−1.30	
4.18	−1.63	1.93	0.87	−0.29	0.48	−3.11	−1.48	−3.24	1.41	0.86	−0.03	
−1.31	−2.12	1.35	−0.13	0.13	0.37	−0.98	0.62	−0.16	−0.23	2.42	−1.83	

layer, which must not exceed 5 mm.

In order to assess the validity of the generated data, the WTR values show insignificant variations between the actual and calculated MR, with maximum differences of up to 0.01 mm, indicating a high level of network reliability under real traffic conditions. According to the information presented, regarding the cracked area values at the end of the design period, Figure 9 provides a more dynamic assessment of significant variations, which were not observed based on the obtained results. It is important to note that the maximum cracked area limit allowed in the MeDiNa method is 30%.

4. Conclusions

In general, the results of the present study indicate that the ANN technique has an excellent capacity for the estimation of the MR values of subgrade soils based on the data available from conventional

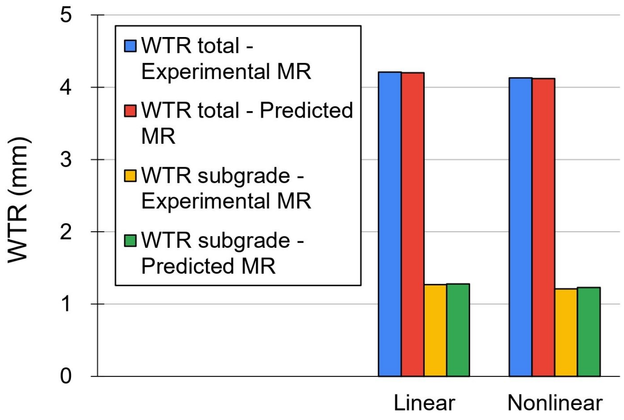

Figure 8 • WTR values for experimental and predicted MR

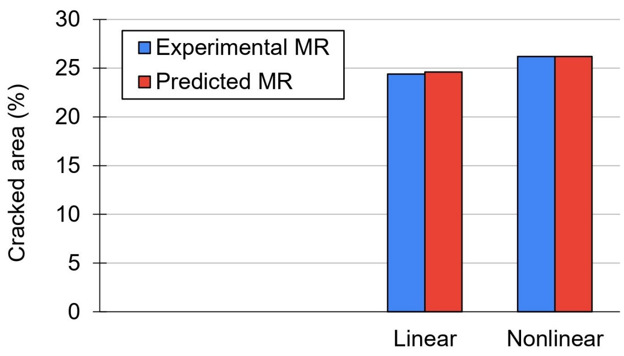

Figure 9 • Cracked area for experimental and predicted MR

surveying of highway subgrades, such as particle size analysis and Proctor compaction tests.

The proposed model proved to be capable of achieving the study objective, demonstrating the ability to predict MR for soils from the state of Ceará. The model therefore represents a low-cost alternative approach, reducing the need for extensive laboratory characterization tests, particularly repeated load triaxial tests, and serving as a useful tool for studies involving low-volume roads.

However, its applicability is conditioned to the geotechnical characteristics of soils from the state of Ceará. Consequently, direct applications at the national scale may lead to inaccuracies due to the regional variability of Brazilian soils. Nevertheless, the regional generalization of the network for Ceará allows the integration of these data into national databases, enabling the development of predictive models applicable at the national level.

This model may be a viable alternative for the triaxial testing of repeated triaxial loads, for the evaluation of the subgrade soils at least for the design of highways with a low volume of road. The procedure described in the present study showed that it is possible to produce models for the rapid and cost-effective prediction of MR values for the design of pavements which can help to minimize the time and financial resources required for the preliminary analysis of subgrade soils for highway pavement design.

Another important contribution of this study is the reinforcement of a methodology capable of reducing laboratory workload, minimizing the need for handling large datasets obtained from experimental tests and simplifying the validation of predicted results. This expands the potential use of replicable computational models in pavement design, while accounting for the regional variability of tropical soils and enabling the regionalization of predictive models, adapting them to the specific geological and climatic conditions of each region.

The processing capability and adaptability of the methodology developed in this study indicate a growing trend toward the use of artificial neural networks to estimate parameters such as MR and PD, particularly in countries facing similar infrastructure challenges and technological limitations. By adopting a methodology based on accessible data, this approach allows countries with limited infrastructure to implement more efficient pavement design solutions without relying on high-cost technologies.

It is nevertheless important to recognize the need for a minimum number of triaxial soil tests in the laboratory to amplify the database, so that new models can be established and updated continuously for beyond the study area.

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CRediT authorship contribution statement

Antonio Júnior Alves Ribeiro: Conceptualization, Methodology, Formal Analysis, Investigation, Data Curation, Writing – Original Draft; Carlos Augusto Uchôa da Silva: Supervision, Methodology, Writing – Review and Editing; Suelly Helena de Araújo Barroso: Supervision, Validation, Writing – Review and Editing; João Paulo Ferreira de Lacerda: Writing – Review and Editing; Pedro Mateus Santos Oliveira: Writing – Review and Editing.

Use of artificial intelligence-assisted technology

The authors state that the methodology used by artificial neural networks was the only technical component of artificial intelligence incorporated into the article.

Competing interests statement

The authors declare that there are no conflicts of interest to report.

Data availability statement

The data, models, and codes that support the findings of this study are available upon request from the corresponding author.

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